C3 **TRIGONOMETRY** *Answers - Worksheet H*

1 **a** $\dfrac{2}{\cos x} = \dfrac{3}{\sin x}$

$\dfrac{\sin x}{\cos x} = \dfrac{3}{2}$

$\tan x = \dfrac{3}{2}$

$x = 56.3, 56.3 - 180$

$x = -123.7°, 56.3°$

b $\cot^2\theta - \cot\theta + 1 + \cot^2\theta = 4$
$2\cot^2\theta - \cot\theta - 3 = 0$
$(2\cot\theta - 3)(\cot\theta + 1) = 0$
$\cot\theta = -1$ or $\dfrac{3}{2}$
$\tan\theta = -1$ or $\dfrac{2}{3}$
$\theta = \pi - \dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}$ or $0.5880, \pi + 0.5880$
$\theta = 0.59$ (2dp), $\dfrac{3\pi}{4}$, 3.73 (2dp), $\dfrac{7\pi}{4}$

2 $2\sin\theta\cos 30 + 2\cos\theta\sin 30$

$\qquad = \sin\theta\cos 30 - \cos\theta\sin 30$

$\sqrt{3}\sin\theta + \cos\theta = \dfrac{\sqrt{3}}{2}\sin\theta - \dfrac{1}{2}\cos\theta$

$\dfrac{\sqrt{3}}{2}\sin\theta = -\dfrac{3}{2}\cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = -\sqrt{3}$

$\tan\theta = -\sqrt{3}$

$\theta = 180 - 60, 360 - 60$
$\theta = 120°, 300°$

3 **a** **i** $\operatorname{cosec} A = \dfrac{1}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$\qquad = \dfrac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$

ii $\operatorname{cosec}^2 A = (2 + \sqrt{3})^2$

$\qquad = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$

$\cot^2 A = \operatorname{cosec}^2 A - 1 = 6 + 4\sqrt{3}$

b $3(1 - 2\sin^2 x) - 8\sin x + 5 = 0$
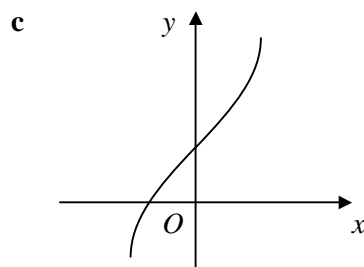$3\sin^2 x + 4\sin x - 4 = 0$
$(3\sin x - 2)(\sin x + 2) = 0$
$\sin x = \dfrac{2}{3}$ or -2 [no solutions]
$x = 41.8, 180 - 41.8$
$x = 41.8°, 138.2°$

4 **a** $= \dfrac{\pi}{2} + 2 \times \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

b $-\dfrac{\pi}{2} \le \mathrm{f}(x) \le \dfrac{3\pi}{2}$

c

d $\dfrac{\pi}{2} + 2\arcsin x = 0$

$\arcsin x = -\dfrac{\pi}{4}$

$x = \sin\left(-\dfrac{\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}$

5 **a** $2 \sin x - 3 \cos x$

$\qquad = R \sin x \cos \alpha - R \cos x \sin \alpha$

$\quad \Rightarrow R \cos \alpha = 2, \ R \sin \alpha = 3$

$\quad \therefore \ R = \sqrt{4+9} = \sqrt{13} = 3.61$

$\qquad \tan \alpha = \frac{3}{2}, \ \alpha = 0.983$

$\quad \therefore \ 2 \sin x - 3 \cos x = 3.61 \sin (x - 0.983)$

b min. value $= -3.61$ (3sf)

$\quad$ when $x - 0.9828 = \frac{3\pi}{2}, \ x = 5.70$ (3sf)

c $\sqrt{13} \sin (2x - 0.9828) + 1 = 0$

$\quad \sin (2x - 0.9828) = -\dfrac{1}{\sqrt{13}}$

$\quad 2x - 0.983 = \pi + 0.2810, \ -0.2810$

$\qquad\qquad\quad = -0.2810, \ 3.4226$

$\quad 2x = 0.7018, \ 4.4054$
$\quad x = 0.35, \ 2.20$

6 **a** $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$

$\quad$ let $A = B = \frac{x}{2}$

$\quad \cos x \equiv \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$

$\quad \cos x \equiv \cos^2 \frac{x}{2} - (1 - \cos^2 \frac{x}{2})$

$\quad \cos x \equiv 2 \cos^2 \frac{x}{2} - 1$

b $\dfrac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{1 + (2 \cos^2 \frac{x}{2} - 1)} = 3 \cot \frac{x}{2}$

$\quad \dfrac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = 3 \cot \frac{x}{2}$
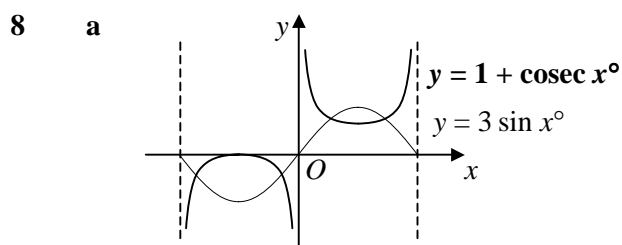
$\quad \tan \frac{x}{2} = \dfrac{3}{\tan \frac{x}{2}}$

$\quad \tan^2 \frac{x}{2} = 3$

$\quad \tan \frac{x}{2} = \pm \sqrt{3}$

$\quad \frac{x}{2} = 60 \ \text{or} \ 180 - 60$

$\quad \frac{x}{2} = 60, \ 120$

$\quad x = 120°, \ 240°$

7 **a** LHS $= \dfrac{1}{\sin \theta} - \sin \theta$

$\qquad = \dfrac{1 - \sin^2 \theta}{\sin \theta}$

$\qquad = \dfrac{\cos^2 \theta}{\sin \theta}$

$\qquad = \cos \theta \times \dfrac{\cos \theta}{\sin \theta}$

$\qquad = \cos \theta \cot \theta$

$\qquad = $ RHS

b $\dfrac{2}{\cos x} + \dfrac{\sin x}{\cos x} = 2 \cos x$

$\quad 2 + \sin x = 2 \cos^2 x$
$\quad 2 + \sin x = 2(1 - \sin^2 x)$
$\quad 2 \sin^2 x + \sin x = 0$
$\quad \sin x (2 \sin x + 1) = 0$
$\quad \sin x = -\frac{1}{2} \ \text{ or } \ 0$

$\quad x = \pi + \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6} \ \text{ or } \ 0, \pi, 2\pi$

$\quad x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

8 **a**

b $3 \sin x = 1 + \dfrac{1}{\sin x}$

$\quad 3 \sin^2 x - \sin x - 1 = 0$

$\quad \sin x = \dfrac{1 \pm \sqrt{1+12}}{6} = \dfrac{1 \pm \sqrt{13}}{6}$

$\quad \sin x = -0.4343 \ \text{ or } \ 0.7676$
$\quad x = -25.7, \ 25.7 - 180 \ \text{ or } \ 50.1, \ 180 - 50.1$
$\quad x = -154.3, \ -25.7, \ 50.1, \ 129.9$

9 **a** LHS $= \sec x + \tan x - \tan x - \sin x \tan x$

$$= \frac{1}{\cos x} - \sin x \times \frac{\sin x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x}$$

$$= \cos x$$

$$= \text{RHS}$$

b $2(1 + \tan^2 2y) + \tan^2 2y = 3$

$\tan^2 2y = \frac{1}{3}$

$\tan 2y = \pm \frac{1}{\sqrt{3}}$

$2y = \frac{\pi}{6}, \pi + \frac{\pi}{6}$ or $\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\quad = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$y = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

10 **a** $4 \sin x - \cos x$

$\quad = R \sin x \cos \alpha - R \cos x \sin \alpha$

$\Rightarrow R \cos \alpha = 4, \; R \sin \alpha = 1$

$\therefore R = \sqrt{16 + 1} = \sqrt{17} = 4.12$

$\tan \alpha = \frac{1}{4}, \; \alpha = 14.0$

$\therefore 4 \sin x° - \cos x° = 4.12 \sin (x - 14.0)°$

b $\frac{2}{\sin x} - \frac{\cos x}{\sin x} + 4 = 0$

$2 - \cos x + 4 \sin x = 0$

$\therefore 4 \sin x° - \cos x° + 2 = 0$

c $\sqrt{17} \sin (x - 14.04) + 2 = 0$

$\sin (x - 14.04) = -\frac{2}{\sqrt{17}}$

$x - 14.04 = 180 + 29.02, 360 - 29.02$

$\qquad = 209.02, 330.98$

$x = 223.1, 345.0 \; (1\text{dp})$

11 **a** adding

$\cos (A + B) + \cos (A - B) \equiv 2 \cos A \cos B$

let $P = A + B$ (1) and $Q = A - B$ (2)

(1) + (2) $\Rightarrow 2A = P + Q \Rightarrow A = \frac{P + Q}{2}$

(1) − (2) $\Rightarrow 2B = P - Q \Rightarrow B = \frac{P - Q}{2}$

$\therefore \cos P + \cos Q \equiv 2 \cos \frac{P + Q}{2} \cos \frac{P - Q}{2}$

b $2 \cos \frac{x + 3x}{2} \cos \frac{x - 3x}{2} + \cos 2x = 0$

$2 \cos 2x \cos (-x) + \cos 2x = 0$

$\cos 2x(2 \cos x + 1) = 0$

$\cos 2x = 0$ or $\cos x = -\frac{1}{2}$

$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$

$\quad$ or $x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}$

12 **a** $3 \cos \theta + 4 \sin \theta$

$\quad = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$

$\Rightarrow R \cos \alpha = 3, \; R \sin \alpha = 4$

$\therefore R = \sqrt{9 + 16} = 5$

$\tan \alpha = \frac{4}{3}, \; \alpha = 0.927 \; (3\text{sf})$

$\therefore 3 \cos \theta + 4 \sin \theta = 5 \cos (\theta - 0.927)$

b **i** $-4 \leq \text{f}(\theta) \leq 6$

$\quad$ **ii** $1 - 5 \cos (2\theta - 0.9273) = 0$

$\cos (2\theta - 0.9273) = \frac{1}{5}$

$2\theta - 0.9273 = 1.3694, 2\pi - 1.3694$

$\qquad = 1.3694, 4.9137$

$2\theta = 2.2967, 5.8410$

$\theta = 1.15, 2.92 \; (2\text{dp})$

c $y = \frac{2}{5\cos(x - 0.9273)}$

TP: $y = \frac{2}{5}$ when $x - 0.9273 = 0$

$\qquad y = -\frac{2}{5}$ when $x - 0.9273 = \pi$

$\therefore (0.93, \frac{2}{5})$ and $(4.07, -\frac{2}{5})$